On Karmakar's Algorithm for Linear Programming

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ABSTRACT: This paper deals with the convergence of the new polynomial-time algorithm developed by N. Karmakar and its relationship with thickness of concentric ellipsoids. Herein, we generalize the Karmakar's inequality to discuss the bounds of objective function and obtain refind form of Karmakar's Formula for the speed of convergence and convergence ration Karmakar's algorithm admits linear convergence in general and supper linear convergence in special case which hopelessly turn out to be trivial. We also show the super linear convergence of the sequence of the values of the objective function at different points of the polytope

Keywords: *linear convergence, linear programming, polynomial-time algorithm, speed of convergence, super linear convergence*

I. Introduction

Karmakar developed a **new polynomial-time algorithm** and developed an inequality to discuss the bond on objective function. We have generalised this inequality and shown there by that smaller the difference between the thickness of pairs of concentric elliposoids. faster the convergence. We have constructed a formula for the speed of convergence and convergence ratio for the algorithm developed admits linear in special case which turn out to be trivial. We have also shown the **super linear convergence** of the sequence of the values of the objective function at different points of the polytope.

II. Relation between Convergence and Thickness of Concentric Ellipsoid

For moving from a point a_k to a_{k+1} in the ellipsoid E_{k+1}, we have inequality

$$\frac{f(a_{k+1})-f_p}{f(a_k)-f_p} \le 1-\frac{1}{v_{k+1}}$$

Multiplication of these (k + 1) inequalities gives

$$\frac{f(a_1)-f_p}{f(a_0)-f_p}\frac{f(a_2)-f_p}{f(a_1)-f_p}\cdots\frac{f(a_{k+1})-f_p}{f(a_k)-f_p} \le \left(1-\frac{1}{v_1}\right)\left(1-\frac{1}{v_2}\right)\cdots\left(1-\frac{1}{v_{k+1}}\right)$$

i.e. $$\frac{f(a_{k+1})-f_p}{f(a_0)-f_p} \le \left(1-\frac{1}{v_1}\right)\left(1-\frac{1}{v_2}\right)\cdots\left(1-\frac{1}{v_{k+1}}\right) \le \left(1-\frac{1}{v}\right)^{k+1}$$

where $\left(1-\frac{1}{v}\right) = \max\left(1-\frac{1}{v_1}\right); 1 \le i \le k+1$

i.e. $$\frac{\lfloor f(a_{k+1}) - f_p \rfloor}{\left[f(a_0) - f_p \right]} \le \mu^{k+1}, \text{ where } \mu = \left(1 - \frac{1}{v}\right) < 1$$

i.e. $$\left[f(a_{k+1}) - f_p \right] \le \mu^{k+1} \left[f(a_0) - f_p \right]$$

Now it follows from the last inequality that $f(a_{k+1}) - f_p \to f_p$ as fast as μ^{k+1} diminishes. But μ^{k+1} diminishes as fast as *v* gets smaller. Hence, smaller the value of *v*. faster the convergence.

Since, $\mu = \max\left(1 - \frac{1}{v_1}\right)\left(1 - \frac{1}{v_2}\right) \ldots \left(1 - \frac{1}{v_{k+1}}\right)$ and also $\mu = \left(\frac{1}{v}\right)$, μ will be minimum if *v* will be minimum. But v will be minimum if each of the factors v_1, v_2 ….. v_{k+1} be minimum. These factors will be minimum only when the difference of thickness of pair of concentric ellipsoid (E_i, E_i); i = 1, 2, …… k + 1, together with $E_i \subset P \subset E_i$; $E_i = v_1 E_i$, be minimum.

Here, Ei be the ellipsoid obtained by magnifying E_i by a suitable large factor v_i to contain the polytope P, and also the thickness of ellipsoid has been used in the same sense.

III. Speed of Convergence and Convergence Ratio

As we have seen, the optimization of *f(x)* over the ellipsoid E_{k+1}, gives the inequality

$$\frac{\left[f(a_{k+1}) - f_p \right]}{\left[f(a_k) - f_p \right]} \le \left(1 - \frac{1}{v_{k+1}}\right) \le \left(1 - \frac{1}{v}\right)$$

since $\left(1 - \frac{1}{v}\right) \max \left(1 - \frac{1}{v_1}\right); 1 \le i \le k+1$

Now $v = (n - 1)$ the diminished of the simplex, can always be achieved by using a suitable projective transformation.

Therefore, the above inequality reduces to

$$\frac{\lfloor f(a_{k+1}) - f_p \rfloor}{\left[f(a_k) - f_p \right]} \le \left(1 - \frac{1}{n-1}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Since the sequence <f(ak)> convergence to f_p, the speed of convergence of the sequence will be given by

$$\lambda = \sup \left\{ q \ge 0 / \lim_{k \to 0} \frac{\left[f(a_{k+1}) - f_p \right]}{\left[f(a_k) - f_p \right]^q} = \beta < \infty \right\}.$$

Here the speed of convergence depends on two parameters: First on the non negative number *q*, and the second, on the convergence ratio β. Thus the large the value of *q*, faster the convergence.

IV. Conclusion

Karmarkar's algorithm revolutionized the field of linear programming by introducing a **Polynomial-time** interior point method that out performs the traditional simplex method for large-scale problems. Unlike the Simplex method, which moves along the edge the edges of the feasible region, Karmarkar's algorithm moves through the **interior** of the feasible Polytope using projective transformations, ensuring significant computational advantage.

Karmakar's algorithm marked a major shift in solving linear programming problems both in terms of theory and practice by offering a faster, more Scalable alternative to existing method.

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